June 2011
Further Pure Mathematics FP3 6669
Mark Scheme

Question Number	Scheme	Marks
1.	$\frac{\mathrm{dy}}{\mathrm{dx}} = 6x^2$ and so surface area $= 2\pi\int 2x^3\sqrt{(1+(6x^2)^2}\,\mathrm{dx}$	B1
	$= 4\pi\left[\frac{2}{3\times 36\times 4}(1+36x^4)^{\frac{3}{2}}\right]$	M1 A1
	Use limits 2 and 0 to give $\frac{4\pi}{216}[13860.016-1] = 806$ (to 3 sf)	DM1 A1
		5
	Notes:	
B1	Both bits CAO but condone lack of 2π	
1M1	Integrating $\int\left(y\sqrt{1+\left(\text{their}\frac{dy}{dx}\right)^2}\right)dx$, getting $k(1+36x^4)^{\frac{3}{2}}$, condone lack of 2π If they use a substitution it must be a complete method.	
1A1	CAO	
2DM1	Correct use of 2 and 0 as limits	
2A1	CAO	
2. **(a) (i)**	$\frac{\mathrm{dy}}{\mathrm{dx}} = \frac{x}{\sqrt{(1-x^2)}} + \arcsin x$	M1 A1 (2)
(ii)	At given value derivative $= \frac{1}{\sqrt{3}} + \frac{\pi}{6} = \frac{2\sqrt{3}+\pi}{6}$	B1 (1)
(b)	$\frac{dy}{dx} = \frac{6e^{2x}}{1+9e^{4x}}$	1M1 A1
	$= \frac{6}{e^{-2x}+9e^{2x}}$	2M1
	$= \frac{3}{\frac{5}{2}(e^{2x}+e^{-2x})+\frac{4}{2}(e^{2x}-e^{-2x})}$	3M1
	$\therefore \frac{dy}{dx} = \frac{3}{5\cosh 2x + 4\sinh 2x}$ *	A1 cso
		(5) **8**
	Notes:	
(a) M1	Differentiating getting an arcsinx term and a $\frac{1}{\sqrt{1\pm x^2}}$ term	
A1	CAO	
B1	CAO any correct form	

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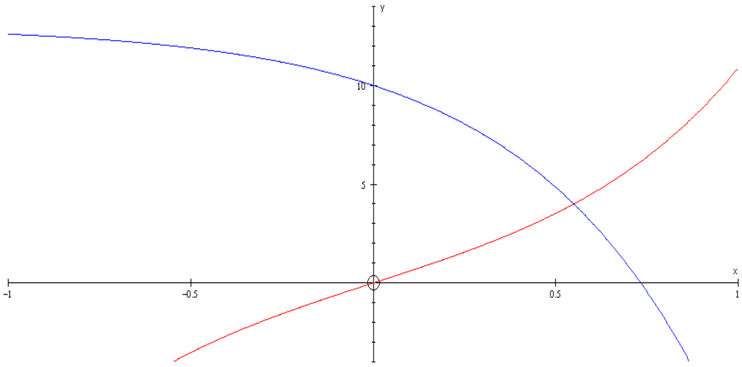
(b) 1M1	Of correct form $\frac{ae^{2x}}{1 \pm be^{4x}}$	
1A1	CAO	
2M1	Getting from expression in e^{4x} to e^{2x} and e^{-2x} only	
3M1	**Using** sinh2x and cosh2x in terms of ($e^{2x}+e^{-2x}$) and ($e^{2x}-e^{-2x}$)	
2A1	CSO – answer given	
3. **(a)**	$x^2-10x+34=(x-5)^2+9$ so $\frac{1}{x^2-10x+34}=\frac{1}{(x-5)^2+9}=\frac{1}{u^2+9}$	B1
	(mark can be earned in either part (a) or (b))	
	$I=\int\frac{1}{u^2+9}du=\left[\frac{1}{3}\arctan\left(\frac{u}{3}\right)\right]$ | $I=\int\frac{1}{(x-5)^2+9}du=\left[\frac{1}{3}\arctan\left(\frac{x-5}{3}\right)\right]$	M1 A1
	Uses limits 3 and 0 to give $\frac{\pi}{12}$ | Uses limits 8 and 5 to give $\frac{\pi}{12}$	DM1 A1
		(5)
(b) Alt 1	$I=\ln\left(\left(\frac{x-5}{3}\right)+\sqrt{\left(\frac{x-5}{3}\right)^2+1}\right)$ or $I=\ln\left(\frac{x-5+\sqrt{(x-5)^2+9}}{3}\right)$	M1 A1
	or $I=\ln\left((x-5)+\sqrt{(x-5)^2+9}\right)$	
	Uses limits 5 and 8 to give $\ln(1+\sqrt{2})$.	DM1 A1
		(4) **9**
(b) Alt 2	Uses u = x-5 to get $I=\int\frac{1}{\sqrt{u^2+9}}du=\left[\operatorname{ar}\sinh\left(\frac{u}{3}\right)\right]=\ln\left\{u+\sqrt{u^2+9}\right\}$	M1 A1
	Uses limits 3 and 0 and ln expression to give $\ln(1+\sqrt{2})$.	DM1 A1
		(4)
(b) Alt 3	Use substitution $x-5=3\tan\theta$, $\frac{dx}{d\theta}=3\sec^2\theta$ and so	M1 A1
	$I=\int\sec\theta d\theta=\ln(\sec\theta+\tan\theta)$	
	Uses limits 0 and $\frac{\pi}{4}$ to get $\ln(1+\sqrt{2})$.	DM1 A1
		(4)
	Notes:	
(a) B1	CAO allow recovery in (b)	
1M1	Integrating getting k arctan term	
1A1	CAO	
2DM1	Correctly using limits.	
2A1	CAO	

Question Number	Scheme	Marks
(b) 1M1	Integrating to get a ln or hyperbolic term	
1A1	CAO	
2DM1	Correctly using limits.	
2A1	CAO	
4.		
(a)	$I_n = \left[\frac{x^3}{3}(\ln x)^n\right] - \int \frac{x^3}{3} \times \frac{n(\ln x)^{n-1}}{x} dx$	M1 A1
	$= \left[\frac{x^3}{3}(\ln x)^n\right]_1^e - \int_1^e \frac{nx^2(\ln x)^{n-1}}{3} dx$	DM1
	$\therefore I_n = \frac{e^3}{3} - \frac{n}{3} I_{n-1}$ *	A1cso
		(4)
(b)	$I_0 = \int_1^e x^2 dx = \left[\frac{x^3}{3}\right]_1^e = \frac{e^3}{3} - \frac{1}{3}$ or $I_1 = \frac{e^3}{3} - \frac{1}{3}\left(\frac{e^3}{3} - \frac{1}{3}\right) = \frac{2e^3}{9} + \frac{1}{9}$	M1 A1
	$I_1 = \frac{e^3}{3} - \frac{1}{3} I_0$, $I_2 = \frac{e^3}{3} - \frac{2}{3} I_1$ and $I_3 = \frac{e^3}{3} - \frac{3}{3} I_2$ so $I_3 = \frac{4e^3}{27} + \frac{2}{27}$	M1 A1
		(4) **8**
	<u>Notes</u>:	
(a)1M1	Using integration by parts, integrating x^2, differentiating $(\ln x)^n$	
1A1	CAO	
2DM1	Correctly using limits 1 and e	
2A1	CSO answer given	
(b)1M1	Evaluating I_0 or I_1 by an attempt to integrate something	
1A1	CAO	
2M1	Finding I_3 (also probably I_1 and I_2) If 'n's left in M0	
2A1	I_3 CAO	

Question Number	Scheme	Marks
5. (a)	 Graph of $y = 3\sinh 2x$	B1
	Shape of $-e^{2x}$ graph	B1
	Asymptote: $y = 13$	B1
	Value 10 on y axis and value 0.7 or $\frac{1}{2}\ln\left(\frac{13}{3}\right)$ on x axis	B1
		(4)
(b)	Use definition $\frac{3}{2}(e^{2x}-e^{-2x}) = 13-3e^{2x} \rightarrow 9e^{4x}-26e^{2x}-3=0$ to form quadratic	M1 A1
	$\therefore e^{2x} = -\frac{1}{9}$ or 3	DM1 A1
	$\therefore x = \frac{1}{2}\ln(3)$	B1
		(5) **9**
	Notes:	
(a) 1B1	y = 3sinh2x first and third quadrant.	
2B1	Shape of y = $-e^{2x}$ correct intersects on positive axes.	
3B1	**Equation** of asymptote, y = 13, given. Penlise 'extra'asymptotes here	
4B1	Intercepts correct both	
(b) 1M1	Getting a three terms quadratic in e^{2x}	
1A1	Correct three term quadratic	
2DM1	Solving for e^{2x}	
2A1	CAO for e^{2x} condone omission of negative value.	
B1	CAO one answer only	

Question Number	Scheme	Marks
6. **(a)**	$\mathbf{n} = (2\mathbf{j} - \mathbf{k}) \times (3\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}) = 6\mathbf{i} - 3\mathbf{j} - 6\mathbf{k}$ o.a.e. (e.g. $2\mathbf{i} - \mathbf{j} - 2\mathbf{k}$)	M1 A1 (2)
(b)	Line l has direction $2\mathbf{i} - 2\mathbf{j} - \mathbf{k}$ Angle between line l and normal is given by $(\cos\beta \text{ or } \sin\alpha) = \frac{4+2+2}{\sqrt{9}\sqrt{9}} = \frac{8}{9}$ $\alpha = 90 - \beta$ = 63 degrees to nearest degree.	B1 M1 A1ft A1 awrt (4)
(c) Alt 1	Plane P has equation $\mathbf{r}.(2\mathbf{i} - \mathbf{j} - 2\mathbf{k}) = 1$ Perpendicular distance is $\frac{1-(-7)}{\sqrt{9}} = \frac{8}{3}$	M1 A1 M1 A1 (4) **10**
(c) Alt 2	Parallel plane through A has equation $\mathbf{r}.\frac{2\mathbf{i} - \mathbf{j} - 2\mathbf{k}}{3} = \frac{-7}{3}$ Plane P has equation $\mathbf{r}.\frac{2\mathbf{i} - \mathbf{j} - 2\mathbf{k}}{3} = \frac{1}{3}$ So O lies between the two and perpendicular distance is $\frac{1}{3} + \frac{7}{3} = \frac{8}{3}$	M1 A1 M1 A1 (4)
(c) Alt 3	Distance A to (3,1,2) = $\sqrt{2^2 + 2^2 + 1^2} = 3$ Perpendicular distance is '3' sin $\alpha = 3 \times \frac{8}{9} = \frac{8}{3}$	M1A1 M1A1 (4)
(c) Alt 4	Finding Cartesian equation of plane P: 2x – y – 2z – 1 = 0 $d = \frac{\lvert n_1\alpha + n_2\beta + n_3\gamma + d \rvert}{\sqrt{n_1^2 + n_2^2 + n_3^2}} = \frac{\lvert 2(1) - 1(3) - 2(3) - 1 \rvert}{\sqrt{2^2 + 1^2 + 2^2}} = \frac{8}{3}$	M1 A1 M1A1 (4)
	Notes:	
(a) M1	Cross product of the correct vectors	
A1	CAO o.e.	
(b) B1	CAO	
M1	Angle between '$2\mathbf{i} - \mathbf{j} - 2\mathbf{k}$' and $2\mathbf{i} - 2\mathbf{j} - \mathbf{k}$, formula of correct form	
1A1ft	8/9ft	
2A1	CAO awrt	
(c) 1M1	Eqn of plane using $2\mathbf{i} - \mathbf{j} - 2\mathbf{k}$ **or** dist of A from O **or** finding length of AP	
1A1	Correct equation (must have =) **or** A to (3,1,2) = 3	
2M1	Using correct method to find perpendicular distance	
2A1	CAO	

Question Number	Scheme	Marks
7. **(a)**	Det $\mathbf{M} = k(0-2)+1(1+3)+1(-2-0) = -2k+4-2 = 2-2k$	M1 A1 (2)
(b)	$\mathbf{M}^T = \begin{pmatrix} k & 1 & 3 \\ -1 & 0 & -2 \\ 1 & -1 & 1 \end{pmatrix}$ so cofactors = $\begin{pmatrix} -2 & -1 & 1 \\ -4 & k-3 & k+1 \\ -2 & 2k-3 & 1 \end{pmatrix}$	M1
	(-1 A mark for each term wrong)	
	$\mathbf{M}^{-1} = \frac{1}{2-2k}\begin{pmatrix} -2 & -1 & 1 \\ -4 & k-3 & k+1 \\ -2 & 2k-3 & 1 \end{pmatrix}$	M1 A3 (5)
(c)	Let (x, y, z) be on l_1. Equation of l_2 can be written as $\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} 4 \\ 1 \\ 7 \end{pmatrix} + \lambda\begin{pmatrix} 4 \\ 1 \\ 3 \end{pmatrix}$.	B1
	Use $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \mathbf{M}^{-1}\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix}$ with $k = 2$. i.e. $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{1}{-2}\begin{pmatrix} -2 & -1 & 1 \\ -4 & -1 & 3 \\ -2 & 1 & 1 \end{pmatrix}\begin{pmatrix} 4+4\lambda \\ 1+\lambda \\ 7+3\lambda \end{pmatrix}$	M1
	$\therefore \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3\lambda+1 \\ 4\lambda-2 \\ 2\lambda \end{pmatrix}$	M1 A1
	and so $(\mathbf{r}-\mathbf{a})\times\mathbf{b} = \mathbf{0}$ where $\mathbf{a} = \mathbf{i} - 2\mathbf{j}$ and $\mathbf{b} = 3\mathbf{i} + 4\mathbf{j} + 2\mathbf{k}$ or equivalent **or** $\mathbf{r} = \mathbf{a} + \lambda\mathbf{b}$ where $\mathbf{a} = \mathbf{i} - 2\mathbf{j}$ and $\mathbf{b} = 3\mathbf{i} + 4\mathbf{j} + 2\mathbf{k}$ or equivalent	B1ft (5) **12**
	Notes:	
(a) M1	Finding determinant at least one component correct.	
A1	CAO	
(b) 1M1	Finding matrix of cofactors or its transpose	
2M1	Finding inverse matrix, 1/(det) cofactors + transpose	
1A1	At least seven terms correct (so at most 2 incorrect) condone missing det	
2A1	At least eight terms correct (so at most 1 incorrect) condone missing det	
3A1	All nine terms correct, condone missing det	
(c) 1B1	Equation of l_2	
1M1	Using inverse transformation matrix correctly	
2M1	Finding general point in terms of λ.	
A1	CAO for general point in terms of one parameter	
2B1	ft for vector equation of their l_1	

Question Number	Scheme	Marks
8. **(a)**	Uses $\frac{dy}{dx}=\frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}=\frac{b\cosh\theta}{a\sinh\theta}$ **or** $\frac{2x}{a^2}-\frac{2yy'}{b^2}=0\rightarrow y'=\frac{xb^2}{ya^2}=\frac{b\cosh\theta}{a\sinh\theta}$	M1 A1
	So $y-b\sinh\theta=\frac{b\cosh\theta}{a\sinh\theta}(x-a\cosh\theta)$	M1
	$\therefore ab(\cosh^2\theta-\sinh^2\theta)=xb\cosh\theta-ya\sinh\theta$ and as $(\cosh^2\theta-\sinh^2\theta)=1$ $xb\cosh\theta-ya\sinh\theta=ab$ *	A1cso
		(4)
(b)	P is the point $(\frac{a}{\cosh\theta},0)$	M1 A1
		(2)
(c)	l_2 has equation $x=a$ and meets l_1 at Q $(a,\frac{b(\cosh\theta-1)}{\sinh\theta})$	M1 A1
		(2)
(d) Alt 1	The mid point of PQ is given by $X=\frac{a(\cosh\theta+1)}{2\cosh\theta}$, $Y=\frac{b(\cosh\theta-1)}{2\sinh\theta}$	1M1 A1ft
	$4Y^2+b^2=b^2\left(\frac{\cosh^2\theta+1-2\cosh\theta+\sinh^2\theta}{\sinh^2\theta}\right)$	2M1
	$=b^2\left(\frac{2\cosh^2\theta-2\cosh\theta}{\sinh^2\theta}\right)$	3M1
	$X(4Y^2+b^2)=ab^2\left(\frac{(\cosh\theta+1)(\cosh\theta-1)2\cosh\theta}{2\cosh\theta\sinh^2\theta}\right)$	4M1
	Simplify fraction by using $\cosh^2\theta-\sinh^2\theta=1$ to give $x(4y^2+b^2)=ab^2$ *	A1cso
		(6)
(d) Alt 2	First line of solution as before	1M1A1ft
	$4Y^2+b^2=b^2\left(\coth^2\theta+\text{cosech}^2\theta-2\coth\theta\,\text{cosech}\theta+1\right)$	2M1
	$=b^2\left(2\coth^2\theta-2\coth\theta\,\text{cosech}\theta\right)$	3M1
	$X(4Y^2+b^2)=ab^2\left(\coth\theta(\coth\theta-\text{cosech}\theta)\right)(1+\text{sech}\theta))$	4M1
	Simplify expansion by using $\coth^2\theta-\text{cosech}^2\theta=1$ to give $x(4y^2+b^2)=ab^2$ *	A1cso
		(6)
		14

Question Number	Scheme	Marks
8.		
(a)1M1	Finding gradient in terms of θ	
1A1	CAO	
2M1	Finding equation of tangent	
2A1	CSO (answer given) look for $\pm(\cosh^2\theta - \sinh^2\theta)$	
(b)M1	Putting $y = 0$ into their tangent	
A1ft	P found, ft for their tangent o.e.	
(c) M1	Putting $x = a$ into their tangent.	
A1	CAO Q found o.e.	
(d)	For Alt 1 and 2	
1M1	Finding expressions, in terms of $\sinh\theta$ and $\cosh\theta$ but must be adding	
1A1	Ft on their P and Q,	
2M1	Finding $4y^2 + b^2$	
3M1	Simplified, factorised, maximum of 2 terms per bracket	
4M1	Finding $x(4y^2 + b^2)$, completely factorised, maximum of 2 terms per bracket	
2A1	CSO	
(d)	For Alts 3, 4 and 5	
1M1	Finding expressions, in terms of $\sinh\theta$ and $\cosh\theta$ but must be adding	
1A1	Ft on their P and Q	
2M1	Getting $\cosh\theta$ in terms of x	
3M1	y or y^2 in terms of $\cosh\theta$ **or** $\sinh\theta$ in terms of x and y	
4M1	Getting equation in terms of x and y only. No square roots.	
2A1	CSO	

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8(d)			
Alt 3	$X=\dfrac{a(\cosh\theta+1)}{2\cosh\theta},\quad Y=\dfrac{b(\cosh\theta-1)}{2\sinh\theta}$	As main scheme	1M1 A1ft
	$\cosh\theta=\dfrac{a}{2x-a}$	$\cosh\theta$ in terms of x	2M1
	$\sinh\theta=\dfrac{b(\cosh\theta-1)}{2y}=\dfrac{b(a-x)}{(2x-a)y}$	$\sinh\theta$ in terms of x and y	3M1
	$\left(\dfrac{a}{2x-a}\right)^2-\left(\dfrac{b(a-x)}{(2x-a)y}\right)^2=1$	Using $\cosh^2\theta-\sinh^2\theta=1$	4M1
	Simplifies to give required equation $\left[y^2 4x(a-x)=b^2(a-x)^2,\ x(4y^2+b^2)=ab^2\right]$		A1cso
			(6)
Alt 4	$X=\dfrac{a(\cosh\theta+1)}{2\cosh\theta},\quad Y=\dfrac{b(\cosh\theta-1)}{2\sinh\theta}$	As main scheme	1M1 A1ft
	$\cosh\theta=\dfrac{a}{2x-a}$	$\cosh\theta$ in terms of x	2M1
	$y^2=\dfrac{b^2(\cosh\theta-1)^2}{4(\cosh^2\theta-1)}=\dfrac{b^2(\cosh\theta-1)}{4(\cosh\theta+1)}$	y^2 in terms of $\cosh\theta$ only	3M1
	$y^2=\dfrac{b^2\left(\dfrac{2a-2x}{2x-a}\right)^2}{4\left(\dfrac{2x}{2x-a}\right)}$ o.e	Forms equation in x and y only	4M1
	Simplifies to give required equation		A1 cso
			(6)
Alt 5	$X=\dfrac{a(\cosh\theta+1)}{2\cosh\theta},\quad Y=\dfrac{b(\cosh\theta-1)}{2\sinh\theta}$	As main scheme	1M1 A1ft
	$\cosh\theta=\dfrac{a}{2x-a}$	$\cosh\theta$ in terms of x	2M1
	$y=\left(\dfrac{b(\cosh\theta-1)}{2\sinh\theta}\right)=\left(\dfrac{b(\cosh\theta-1)}{2\sqrt{\cosh^2\theta-1}}\right)$	y in terms of $\cosh\theta$ only	3M1
	Eliminate $\sqrt{\ }$ and forms equation in x and y		4M1
	Simplifies to give required equation		A1cso